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G. P. Barker<sup>†</sup>, A. Berman<sup>\*</sup> and R. J. Plemmons<sup>\*\*</sup>

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ABSTRACT

We study various stability type conditions on a matrix  $A$  related to existence of a positive diagonal matrix  $D$  such that the Lyapunov matrix  $Q = AD + DA^t$  is positive definite. Such problems arise in mathematical economics, in the study of time-invariant continuous-time systems and in the study of predator-prey systems. Using a theorem of the alternative, a characterization is given for all  $A$  such that a corresponding  $D$  exists. In addition, some necessary conditions for consistency and some related ideas are discussed. Finally, a method for constructing  $D$  is given for matrices  $A$  satisfying certain conditions.

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# POSITIVE DIAGONAL SOLUTIONS TO THE LYAPUNOV EQUATIONS

G. P. Barker<sup>†</sup>, A. Berman<sup>\*</sup> and R. J. Plemmons<sup>\*\*</sup>

All the vectors in this paper are real and the matrices are square and real.

The well known theorem of Lyapunov [1892] states that all the eigenvalues of a matrix  $A$  have negative (positive) real parts if and only if there exists a symmetric positive definite matrix  $H$  such that

$$Q = AH + HA^t$$

is negative (positive) definite. Such a matrix  $A$  is said to be negative (positive) stable and such matrices have been studied extensively in the economics and mathematics literatures (e.g. Johnson [1974] and Taussky [1961]). By a stable matrix we shall mean a positive stable matrix.

We are concerned here with conditions under which the matrix  $H$  in the expression for  $Q$  can be chosen to be a positive diagonal matrix  $D$ . When this is the case,  $A$  is called diagonally stable.

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Matrices  $A$  for which such a matrix  $D$  exists are important in the study of dynamical systems (e.g. Araki [1975]), in the study of predator-prey systems (e.g. Krikorian [1976]) and in the study of dynamic equilibria in economics (e.g. Johnson [1974]).

We first collect some observations.

Lemma 1. The following statements are equivalent for a matrix  $A$ .

(1) There exists a positive diagonal matrix  $D$  such that

$$Q = AD + DA^t$$

is positive definite.

(1<sub>a</sub>) There exists a positive diagonal matrix  $D$  such that

$$x^t ADx > 0$$

for all nonzero vectors  $x$ .

(1<sub>b</sub>) There exists a positive diagonal matrix  $D$  such that the symmetric part of

$$D^{-\frac{1}{2}}AD^{\frac{1}{2}}$$

is positive definite.

(1<sub>c</sub>)  $A^t$  satisfies Condition (1).

(1<sub>d</sub>)  $A^{-1}$  exists and satisfies Condition (1).

(1<sub>e</sub>) For each positive diagonal matrix  $E$ ,  $AE$  and  $EA$  satisfy Condition (1).

(1<sub>f</sub>) Every principal submatrix of  $A$  satisfies Condition (1).

Proof. Clearly Conditions (1) and  $(1_a)$  are equivalent. To complete the proof it is enough to show that (1) implies  $(1_b)$ ,  $(1_c)$ ,  $(1_d)$ ,  $(1_e)$  and  $(1_f)$ .

Let  $D$  be a positive diagonal matrix such that  $AD + DA^t = P$  is positive definite. Then

$$D^{-\frac{1}{2}}AD^{\frac{1}{2}} + (D^{-\frac{1}{2}}AD^{\frac{1}{2}})^t = D^{-\frac{1}{2}}(AD + DA^t)D^{-\frac{1}{2}} = D^{-\frac{1}{2}}PD^{-\frac{1}{2}}$$

which is positive definite, so that  $(1_b)$  holds. Clearly  $(1_c)$  follows since  $P^t$  is positive definite. Also  $A$  is nonsingular, being stable, and

$$A^{-1}D + D(A^{-1})^t = A^{-1}P(A^{-1})^t$$

which is positive definite so that  $(1_d)$  holds. If  $E$  is a positive diagonal matrix then

$$(AE)(E^{-1}D) + (E^{-1}D)(AE)^t = P,$$

$$(EA)(DE) + (DE)(EA)^t = EPE$$

so that  $(1_e)$  holds. The statement  $(1) \Rightarrow (1_f)$  follows from the fact that ~~principal~~ submatrices of a positive definite matrix are also positive definite. ■

We now characterize these matrices utilizing the theory of the alternative for systems of linear inequalities.

Theorem 1. A matrix  $A$  is diagonally stable if and only if for every nonzero positive semidefinite matrix  $B$ ,  $BA$  has a positive diagonal element.

Proof. Let  $V_1$  be the space of real diagonal matrices ordered by the cone  $K_1$  of nonnegative diagonal matrices. Let  $V_2$

be the space of symmetric matrices ordered by the cone  $K_2$  of positive semi-definite matrices. In both cases we use the inner product

$$(A, B) = \text{tr}(AB) .$$

If the dual cone  $K^*$  of  $K \subseteq V$ , is defined by

$$K^* = \{B \in V; (B, A) \geq 0 \text{ for all } A \in K\}$$

then  $K_1^* = K_1$  and  $K_2^* = K_2$  (cf. Hall [1967]). Define the linear operator  $T_A: V_1 \rightarrow V_2$  by

$$T_A(D) = AD + DA^t .$$

The theorem of the alternative (Berman and Ben-Israel [1971], Berman [1973], p. 20) says that the system

$$T_A(D) \in \text{int } K_2, \quad D \in \text{int } K_1$$

is consistent if and only if there is no nonzero solution for the system

$$-T_A^t(B) \in K_1^*, \quad B \in K_2^* .$$

The mapping  $T_A^t$  is defined by

$$(T_A^t(B), D) = (B, T_A(D))$$

for all  $D \in V_1$ . Thus  $T_A^t(B)$  is a diagonal matrix which satisfies

$$\text{tr}(D_A^t(B)D) = \text{tr}(BT_A(D)) = \text{tr}(B(AD + DA^t)) = \text{tr}((BA + A^tB)D)$$

for every diagonal matrix  $D$ . Thus

$$T_A^t(B) = \text{diag } (BA + A^tB)_{ii} = 2 \text{diag } (BA)_{ii} .$$

The conclusion follows. ■

Similar results can be obtained for the operators defined by Stein [1952], Schneider [1965] and Hill [1969]. For the Stein operator  $C_A$  we have

$$C_A(D) = D - A^t D A \quad \text{and} \quad C_A^t(B) = \text{diag}\{B_{ii} - (ABA^t)_{ii}\}.$$

We now list some immediate corollaries to Theorem 1. The first corollary and the "only if" part of the second corollary can also be found in Johnson [1974].

Corollary 1. If  $A$  is a triangular matrix with positive diagonal elements, then  $A$  satisfies (1).

Proof. By Lemma 1,  $A$  can be assumed to be upper triangular. Let  $k$  be the smallest index such that for a given positive semidefinite matrix  $B$ ,  $b_{kk} > 0$ . Then  $(BA)_{kk} > 0$ . ■

Corollary 2. A matrix  $A$  of order 2 is diagonally stable if and only if all its principal minors are positive.

Proof. The "only if" part holds for matrices of any order as we shall show in the next theorem. It is enough to prove the "if" statement for matrices  $A$  of the form

$$A = \begin{pmatrix} 1 & a_{12} \\ a_{21} & 1 \end{pmatrix}.$$

We must show that for every nonzero positive semidefinite  $B$ , either  $(BA)_{11} > 0$  or  $(BA)_{22} > 0$ . If  $b_{ii} = 0$ , then  $(BA)_{ii} > 0$ . If  $b_{11} > 0$  and  $b_{22} > 0$  we may assume that

$$B = \begin{pmatrix} 1 & b \\ b & c \end{pmatrix}$$

with  $0 < b^2 \leq c$ . We also have  $1 > a_{12}a_{21}$ , whence  $c > a_{12}a_{21}b^2$ .

If now

$$(BA)_{11} = 1 + a_{21}b \leq 0 \text{ and}$$

$$(BA)_{22} = c + a_{12}b \leq 0, \text{ then}$$

$$c \leq (-a_{21}b)(-a_{12}b) = a_{21}a_{12}b^2, \text{ a contradiction.} \quad \blacksquare$$

Some fairly obvious necessary conditions for a matrix to satisfy Condition (1) are now given.

Theorem 2. If a matrix  $A$  satisfies Condition (1) of Lemma 1 then it also satisfies each of the following conditions.

- (2) All the principal submatrices of  $A$  are stable.
- (2<sub>a</sub>) The real part of each eigenvalue of each principal submatrix of  $A$  is positive.
- (3) All the principal minors of  $A$  are positive.
- (3<sub>a</sub>) Every real eigenvalue of each principal submatrix of  $A$  is positive.
- (3<sub>b</sub>)  $A + E$  is nonsingular for each nonnegative diagonal matrix  $E$ .

(3<sub>c</sub>) For each  $x \neq 0$  there is a positive diagonal matrix  $E$  such that

$$x^t A E x > 0 .$$

(3<sub>d</sub>)  $A$  does not reverse the sign of any vector. That is, if  $x \neq 0$

then for some subscript  $i$

$$x_i (Ax)_i > 0 .$$

(3<sub>e</sub>) For each signature matrix  $S = \text{diag}(\pm 1)$ , there exists an  $x > 0$  such that

$$S A S x > 0 .$$

Proof. It is well known that Conditions (2) and (2<sub>a</sub>) are equivalent by the theorem of Lyapunov [1892] for arbitrary matrices  $A$ . Moreover Conditions (3) through (3<sub>e</sub>) are also equivalent (cf. Plemmons [1976]) and (2<sub>a</sub>) implies (3). Also, it is clear that (1) implies (2) by Condition (1<sub>e</sub>). ■

For the next result we need the following notation. Let

$\langle n \rangle = \{1, \dots, n\}$ . For any  $n \times n$  matrix  $C$  and any  $\alpha \subseteq \langle n \rangle$ , let  $C[\phi] = 1$ , and for  $\alpha \neq \phi$  let  $C[\alpha]$  be the principal submatrix of  $C$  with indices from  $\alpha$  and let  $C(\alpha)$  be the principal submatrix of  $C$  with indices from  $\langle n \rangle - \alpha$ . If  $D = C^{-1}$  then from Sylvester's identity, (cf. Gantmacher and Krein [1960], p. 15) we have

$$(*) \quad \det D[\alpha] = (\det C)^{-1} \det C(\alpha) .$$

Theorem 3. If a matrix  $A$  satisfies any of Conditions (1), (2) or (3) in Theorem 2, then  $A$  is nonsingular and each of the matrices  $A^t$  and  $A^{-1}$  satisfies the same condition.

Proof. The only nonimmediate assertion is that if  $A$  satisfies (2) then so does  $A^{-1}$ . Suppose  $A$  satisfies (2). Let  $z$  be any complex number such that  $\operatorname{Re} z \geq 0$  and put

$$A_z = A + zI, \quad B_z = A_z^{-1}.$$

Clearly every principal submatrix of  $A$  is stable. Then for any  $\alpha \subseteq \langle n \rangle$  we have from (\*) that

$$\det B_z[\alpha] = (\det A_z)^{-1} \det A_z(\alpha).$$

But the stability of  $A_z(\alpha)$  yields that the right hand side is nonzero.

Since  $z$  is any number in the closed right half plane it follows that

$B[\alpha]$  has no eigenvalues in the closed left half plane, i.e.  $B$  satisfies (2). ■

For a given  $A$  we have the following result on the inductive construction of a  $D$  satisfying (1).

Theorem 4. Let  $A$  be partitioned  $(n-1, 1)$  so that

$$A = \begin{pmatrix} A_1 & a_2 \\ a_3^T & a \end{pmatrix}.$$

Suppose there is a positive diagonal matrix  $D$ , of order  $n-1$  such that

$A_1 D_1 + D_1 A_1^t = A_{11}$  is positive definite. If  $z = (z_i)$  is a vector of order  $n-1$ , define the inner product  $\langle z_1 | z_2 \rangle$  by

$$\langle z_1 | z_2 \rangle = z_1^T A_{11}^{-1} z_2.$$

Let  $x = a_2$ ,  $y = D_1 a_3$ . Then a positive scalar  $d$  exists such that for

$$D = \begin{bmatrix} D_1 & 0 \\ 0 & d \end{bmatrix}, \quad AD + DA^t \text{ is positive definite, if and only if} \\ a > (\langle x|y \rangle + \langle x|x \rangle \langle y|y \rangle).$$

Proof. Regarding  $d$  as an unknown we note that

$$AD + DA^t = \begin{bmatrix} A_1 D_1 + D_1 A_1^t & a_2 d + D_1 a_3 \\ a_3^t D_1 + d a_2^t & 2ad \end{bmatrix} \text{ is positive definite if and only}$$

if its determinant  $\Delta$  is positive. But  $\Delta = (\det A_{11})[(2da - a_3^t D_1 + d a_2^t) A_{11}^{-1}(a_2 d + D_1 a_3)] > 0$  when and only when

$$0 < 2da - a_3^t D_1 A_{11}^{-1} D_1 a_3 - a_3^t D_1 A_{11}^{-1} a_2 d - d a_2^t A_{11}^{-1} D_1 a_3 - d a_2^t A_{11}^{-1} a_2 d^2 \\ = 2da - \langle y|y \rangle - 2\langle y|x \rangle d - \langle x|x \rangle d^2.$$

Put  $p(d) = \langle x|x \rangle d^2 + (2\langle x|y \rangle - 2a)d + \langle y|y \rangle$ . There is a  $d$  for which  $p(d) < 0$  if and only if for the discriminant of  $p$  we have

$$0 < 4\{(\langle x|y \rangle - a)^2 - \langle x|x \rangle \langle y|y \rangle\}.$$

Put  $q(z) = z^2 - 2\langle x|y \rangle z + (\langle x|y \rangle^2 - \langle x|x \rangle \langle y|y \rangle)$ . We have

$q'(z) = 2z - 2\langle x|y \rangle$ , so at  $z = \langle x|y \rangle$ ,  $q(z)$  has the negative minimum  $q(\langle x|y \rangle) = -\langle x|x \rangle \langle y|y \rangle$ . The larger root of  $q(z) = 0$  is  $z = \langle x|y \rangle + \langle x|x \rangle \langle y|y \rangle$ . So for  $a > a_0 = \langle x|y \rangle + \langle x|x \rangle \langle y|y \rangle$  the discriminant of  $p$  is positive. In order that a  $d > 0$  exists we must have that the larger root of  $p(d) = 0$  is positive. However the larger root of  $p(d) = 0$  is given by

$$r(a) = a - \langle x|y \rangle + (\langle x|y \rangle - a)^2 - \langle x|x \rangle \langle y|y \rangle.$$

We see that  $r(a_0) > 0$  and  $r(a)$  is increasing for  $a > a_0$ . Hence for any  $a > a_0$  the discriminant of  $p$  is positive and the larger root is positive. Thus we can find the required  $d$ . ■

Remarks.

1. If  $A$  satisfies any of Conditions (1), (2) or (3) then so does each principal submatrix of  $A$ .
2. If  $A$  satisfies Condition (1) of Lemma 1, then there need not always exist a positive diagonal matrix  $D$  such that

$$AD + DA^t = I.$$

In fact, the proof of Theorem 3 in Taussky [1961] shows that such a  $D$  exists if and only if

$$A = E + SE$$

where  $E$  is a positive diagonal matrix and  $S$  is skew symmetric.

3. It is easy to construct matrices which satisfy (3) but not (2) of Theorem 2 and thus not (1). For example, the matrix

$$A = \begin{pmatrix} 3 & 4 & 2 \\ 2 & 3 & 4 \\ 4 & 2 & 3 \end{pmatrix}$$

satisfies (3), but its eigenvalues are  $9$  and  $\pm\sqrt{3}i$  so that it is not stable and thus does not satisfy (2). Note also that  $A$  is normal.

4. To show that (2) does not imply (1) we proceed as follows: Let  $A$  be the matrix given in Remark 3. Let

$$D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 + \epsilon \end{pmatrix},$$

with  $\epsilon > 0$ . Then for sufficiently small  $\epsilon$ , all the principal submatrices of  $AD$  are stable. Thus  $AD$  satisfies Condition (2), but by Remark 3 and Lemma 1,  $AD$  does not satisfy Condition (1).

5. The matrices  $A = (a_{ij})$  arising in the work of Krikorian [1976], mentioned earlier, satisfy  $a_{ij}a_{ji} \geq 0$  for all  $i \neq j$ . The example in Remark 4 shows that even for these matrices Condition (2) does not imply Condition (1).

6. For certain classes of matrices, Conditions (1), (2) and (3) are all equivalent. These classes include:

- (a) The triangular matrices.
- (b) The positive definite matrices.
- (c) The  $2 \times 2$  matrices.
- (d) The matrices  $A = (a_{ij})$  with  $a_{ij} \leq 0$  for  $i \neq j$ .

The matrices satisfying (d) and (1), (2) or (3) are of course the non-singular M-matrices.

7. A significantly large class of matrices which satisfy (1) can be obtained as follows: For  $A = (a_{ij})$ , let  $\mathcal{M}(A)$  be the matrix given by

$$m_{ij} = \begin{cases} |a_{ij}| & \text{if } i = j \\ -|a_{ij}| & \text{if } i \neq j \end{cases}.$$

Then it is known (e.g. Plemmons [1976]) that if  $A$  has a positive diagonal then each of the following conditions is equivalent:

- (a)  $\mathcal{M}(A)$  is a nonsingular  $M$ -matrix.
- (b) There exists  $x > 0$  such that for each signature matrix  $S = \text{diag}(\pm 1)$ ,

$$SASx > 0.$$

- (c) There exists a positive diagonal matrix  $D$  such that  $AD$  is strictly row diagonally dominant.

- (d) There exists a positive diagonal matrix  $D$  such that

$$D^{-1}AD$$

is strictly row diagonally dominant.

It follows from the work of Johnson [1974] that any matrix  $A$  with a positive diagonal, satisfying any of these conditions, also satisfies (1). That the converse is not true is easily established. Let

$$A = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}.$$

Then by Corollary 2,  $A$  satisfies (1). But  $\mathcal{M}(A)$  is a singular matrix.

#### Open Question

A matrix  $A$  was defined by Arrow and McManus [1958] to be  $D$ -stable if  $A$  satisfies:

(4) For each  $D = \text{diag}(d_1, \dots, d_n)$ ,  $AD$  is stable if and only if  $d_i > 0$ ,  $i = 1, \dots, n$ .

The more common practice now is to define  $A$  to be  $D$ -stable if it satisfies

(5)  $AD$  is stable for each positive diagonal matrix  $D$ .

The class of matrices satisfying (5) properly contains the class of matrices satisfying (4) which, in turn, properly contains the class satisfying (1). That these classes are not the same is illustrated by the matrix

$$A = \begin{pmatrix} 0 & -1 \\ 1 & 1 \end{pmatrix}$$

which satisfies (4), but not (1). Examples also exist that satisfy (5) but not (4) (e.g. Johnson [1974]).

In view of these conditions we pose the following open question:

Are the topological interiors of the sets of matrices satisfying (1), (4) and (5) identical?

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